July 15th, 2011

Don't forget to indicate your name. Explain your reasoning - you may get some points for the intermediate steps even if you fail to solve the problem completely. For 2 point problems it is compulsory.

Thermodynamics

1. The temperature of a heat generator of a modern power station, which is using supersaturated vapor as a working substance, is 600° C. As a cooler, water from a nearby river with a temperature of 20° C is used. Find the maximal efficiency of the power station. (**2 points**)

2. Prove that the heat capacity at constant volume C_V of van-der-Waals gases does not depend on the volume. (**4 points**)

3. Two identical reservoirs A and B with volumes V_0 contain (A) one-atomic and (B) two-atomic gases at pressures P_0. Both reservoirs are instantly compressed adiabatically to the half of the initial volume. Find the ratio W_A/W_B of the works required to compress the gases A and B. (**4 points**)

4. Find the pressure of an air as a function of altitude h if the temperature of the air decreases linearly with h, i.e., the temperature gradient is constant and is equal to $-a$ ($dT/dh = -a$). Express also the pressure as a function of temperature under this condition. From the results obtained find the limiting equation for an isothermal atmosphere ($a \to 0$). In your derivations assume that the air can reasonably be described by the ideal gas law. (**6 points**)

Electrostatics

1. Three identical batteries with the electromotive forces ε are connected as shown in Figure 1. What is the potential difference ΔV_{AB} between the points A and B? (**2 points**)

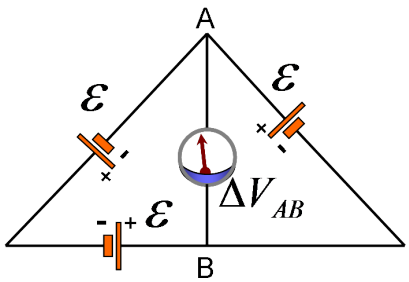

Figure 1: An electric circuit.

2. The Earth is continuously bombarded by cosmic rays of high energy. Using measurements performed on satellites, it was established that they are predominantly composed of protons. The average energy $< E_p >$ of these protons was found to be of the order of a few 10^9 eV (1 eV $\approx 1.6 \times 10^{-19}$ J). The radiation intensity was found to be about 1 proton per square centimeter per second. Assume that initially the Earth had no charge and each proton colliding with the Earth surface is absorbed, i.e. contributes to the increase of the total charge of the Earth. Estimate the order of magnitude value of the time required for the cosmic rays to increase the electric potential of the Earth to such a value that the protons could not anymore reach its surface due to the electrostatic repulsion. The Earth radius is $R_{Earth} \approx 6350$ km. (**4 points**)

3. A rectangular frame (the short and long side lengths are a and $2a$, respectively) is made of a thin metal wire and has a total mass m. The frame can freely (without any friction) rotate around the vertical axis passing through the centers of two opposite sides of the frame as shown in Figure (left panel). The frame carries an electric current I. When a horizontal magnetic field is applied, the frame assumes the equilibrium position, characterized by some angle θ_0 between the magnetic field and the normal to the frame plane. Given a small deviation from θ_0 what will be the period of oscillations of the frame? (**4 points**)

4. Using a velocity selector and a slit with the opening width $d_0 = 0.1$ cm, a narrow beam of electrons with the kinetic energy $E_k = 400$ eV is selected as shown in Figure (right panel). Find the distance x form the slit where the beam thickness d will be doubled ($d = 2d_0$) due to the electrostatic interaction. The electric current per unit length j in z-direction (see figure) in the slit is 10^{-4} A/cm. (**6 points**)

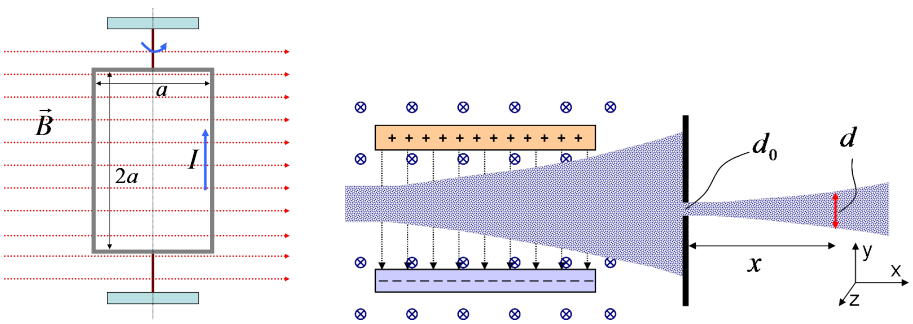

Figure 2: (Left panel) A freely rotating metal frame in the magnetic field. (Right panel) Top view of a velocity selector.

The total number of points: **32 points**
A rough conversion scale: 4 - 9P; 3 - 14P; 2 - 20P; 1 - 27P