I will start by discussing some of the main challenges that one faces when trying to formulate a nonequilibrium thermodynamics of complex systems. I will then discuss how to use stochastic thermodynamics to make progress in that direction. Concrete evidence will be provided by considering applications to energy transduction in (bio-) chemical reaction networks, low power computation with nonlinear electronic circuits, and nonequilibrium phase transitions in Potts models.